

Research paper

# Evaluation of the Applicability of the Weibull Probability Density Function with Different Approaches for Diameter Distributions of Plantations of Four Conifer Species

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## [ Summary ]

The purpose of this research was to quantify diameter distributions of stands using the Weibull probability density function for 4 conifers, namely China fir (*Cunninghamia lanceolata*), Japanese cedar (*Cryptomeria japonica*), Taiwan red cypress (*Chamaecyparis formosensis*), and Taiwania (*Taiwania cryptomerioides*). These 4 conifers are widely planted in mountain areas of Taiwan, and we focused on plantation forests. The study area included all of Taiwan, and the data were from a system of permanent sample plots of the National Forest Management Plan Revision. We utilized the Weibull function to model diameter distributions based on 3 approaches of the maximum likelihood estimator (MLE) and 2 percentile estimators [PE(I) and PE(II)]. After examining the goodness-of-fit for the observed data and the Weibull distribution estimated using the 3 approaches with the Kolmogorov-Smirnov (K-S) test, we found that PE(II) was superior to the other approaches, and the respective pass rates of samples were 100, 96.6, 85.5, and 97.2% for China fir, Japanese cedar, Taiwan red cypress and Taiwania. A similar trend of pass rates of samples indicated that PE(II) > PE(I) > MLE for each species. The results implied that PE(II) was the best estimator among the 3 approaches for these 4 conifers.

**Key words:** Weibull probability density function, conifer plantations, maximum likelihood estimators, percentile estimators.

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## 研究報告

## 評估不同方法推估之Weibull函數在四種針葉樹 人工林直徑分布之適用性

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### 摘要

本研究旨在探討應用Weibull機率密度函數應用於臺灣四種主要針葉樹種—杉木、柳杉、紅檜及臺灣杉林分直徑分布之適用性。此四種針葉樹過去廣泛地栽植於臺灣各地區，本研究主要以此四種針葉樹人工林為研究對象，研究區域分布於整個臺灣，所採用之資料主要來自於國有林事業區森林經營計畫檢訂調查之永久樣區系統，林分直徑分布建模係採用最大概似法(the maximum likelihood estimator, MLE)和兩種百分位數法[the percentile estimators, PE(I) and PE(II)]配適Weibull機率密度函數，建模後以Kolmogorov-Smirnov測驗檢測觀測值和由三種方法模擬所得之Weibull分布之適合度，所得結果以PE(II)模擬結果最佳，其樣本通過檢測比例在杉木、柳杉、紅檜及臺灣杉分別為100、96.6、85.5及97.2%。此外研究結果也發現三種推估方法在模擬不同樹種呈現一致性趨勢，即樣本通過檢測比例： $PE(II) > PE(I) > MLE$ ，此結果顯示PE(II)之推估結果最適合四種樹種之直徑分布。

關鍵詞：Weibull機率密度函數、針葉樹人工林、最大概似法、百分位數法。

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## INTRODUCTION

Diameter at breast height (DBH) is one of most important variables for measuring trees and is usually regarded as a standard variable to present tree diameter because DBH is easy to measure and is well correlated with other tree characteristics, such as tree height, crown width, tree volume, biomass (foliage, branches, and stems), and carbon storage (Hush et al. 1982, Avery and Burkhart 1983, Clutter et al. 1983, Yen et al. 2009). Since DBH is a standard measure of tree diameter, the stand diameter distribution implies DBH distributions within stands. It is a valid expression to show tree size distributions within stands (Bailey and Dell 1973, Cao et al. 1982, Clutter et al. 1983, Davis et al. 2001, Cao 2004, Yen et al. 2010). Quan-

tifying diameter distributions is an important task for making decisions by forest managers; it can be correlated with stand growth, stand density management, and economic and ecological values of forests (Bailey and Dell 1973, Clutter et al. 1983, Davis et al. 2001).

Quantifying diameter distributions of stands usually adopts the probability density function (PDF) as a model, and many well-known PDFs have been proposed to fit stand diameter distributions, such as the gamma distribution (Nelson 1964), beta distribution (Clutter and Bennett 1965), and Weibull distribution (Bailey and Dell 1973). Among these PDFs, the Weibull function is widely utilized to characterize various forests because it possesses many advantages, including that the

model is flexible and can fit various diameter distributions, the parameters have geometrical meanings, and its cumulative density function (CDF) can be easily converted from the PDF (Bailey and Dell 1973, Cao et al. 1982, Hyink and Moser 1983, Yen 1999a, b, Yen et al. 2010). We also found that many research reports developed diameter distributions for different forest types using the Weibull function, including natural forests (Lee and Chen 1991, Lee and Jong 1996), planted forests (Yen 1999a, b, Li and Yen 2010), and bamboo forests (Yen et al. 2010). Their results indicated that this model possesses an excellent ability to describe various forest types. On the other hand, the parameters of the Weibull function can be estimated using various approaches, e.g., maximum likelihood estimators (MLEs) and percentile estimators (PEs). Interestingly, we also found that these 2 approaches have individual advantages, and might be suitable for different forest types (Bailey 1974, Abernethy 1981, Cao et al. 1982, Clutter et al. 1983, Zarnoch and Dell 1985, Nanang 1998, Yen 1999a, b, Yen et al. 2010). Although many research reports fit diameter distributions of stands with the Weibull function in Taiwan, most such research was limited to a regional area, and a large-scale scope of all of Taiwan was rarely addressed.

The importance of plantations arises from current forest management practices in Taiwan because many limits are imposed on natural forests, such as emphasizing conservation and prohibiting cutting for commercial uses (Yen et al. 2013). Compared to natural forests, many environmental, ecological, and economic goals can be widely programmed in plantation forests. Traditional targets of man-made forest management focused on timber production, and in consequence, China fir (*Cunninghamia lanceolata*), Japanese cedar (*Cryptomeria japonica*), Taiwan red cypress

(*Chamaecyparis formosensis*), and Taiwania (*Taiwania cryptomerioides*) are 4 important conifers with a large number of plantations in Taiwan (Huang et al. 2011). Numerous research reports studied the growth and yield for these 4 conifers because they possess excellent wood properties, high productivity, and commercial value (e.g., Hwang 1977, Chiu and Lo-Cho 2002, Yen et al. 2009, Huang et al. 2011). Beyond traditional timber production, we also found that many studies focused on assessing carbon sequestration by these conifers due to their high productivity (e.g., Lin et al. 2003, Yen et al. 2004, 2009, Yen and Huang 2006, Yen and Wang 2013). On the other hand, most growth and yield studies focused on regional areas for these 4 conifers, and few addressed a larger scale.

This study focused on these 4 major conifer plantations distributed throughout Taiwan. Most man-made forests result from planting forests, and this is a common phenomenon in Taiwan with plantation forests covering approximately 420,000 ha in Taiwan (TFB 1995, Yen et al. 2013). Moreover, this forest type plays an important role in timber production, environmental protection, carbon storage, and other ecological services (Yen and Wang 2013). Quantifying diameter distributions of stands can help assess the current status and future development of forests. Therefore, it is an important task for managing plantation forests.

This study developed diameter distributions of plantations of 4 major Taiwanese conifer species with the Weibull function based on different approaches. The purposes of this study were to: (1) fit diameter distributions of stands with the Weibull function based on the MLE and PE methods, (2) compare the estimation ability between the MLE and PE methods, and (3) assess the applicability of the Weibull function among these 4 species.

## MATERIALS AND METHODS

### Study areas

The study was conducted in plantations of 4 major Taiwanese conifer species, namely China fir, Japanese cedar, Taiwan red cypress, and Taiwania. The study area was located in Taiwan (22°30'N to 24°51'N, 120°39'E to 121°42'E) and at elevations of 410~2680 m. These 4 conifers are dominant in man-made forests that occupy a broad area in Taiwan. Data on these plantations were obtained from the National Forest Management Plan Revision (NFMPR) of Taiwan, and all plantations belong to the Taiwan Forestry Bureau (TFB). The NFMPR is an important task for decision-making in forest management because it provides current growing stocks of forests for the TFB. The NFMPR is based on a system of permanent sample plots (PSPs) that contains 3188 PSPs distributed throughout Taiwan and covers most important forest types, including natural forests and man-made forests (Huang et al. 2011, TFB 2014). The system of PSPs was established by the TFB in 1997, and the first survey was completed in 2002 (Huang et al. 2011). The PSP has 2 main plot-sizes that contain 0.02 and 0.05 ha for man-made forests. As the stands consist of small trees and tree sizes are homogeneous, the PSP was suggested to adopt a small size (0.02 ha); otherwise, a large PSP (0.05 ha) was adopted (Huang et al. 2011, TFB 2014). Records of the PSP survey contain detailed measures of DBH and tree height for each tree, and the PSP is repeatedly surveyed every 5 yr. The second survey was also completed, and the data of these 2 periods were obtained in this study. Because only a slight difference in the diameter distributions was shown between the 2 periods in each PSP, we selected the latter survey as samples for our analysis.

This study extracted data from 18, 88,

55, and 36 PSPs for China fir, Japanese cedar, Taiwan red cypress, and Taiwania, respectively. Detailed distributions of the PSPs of the 4 conifers are shown in Fig. 1.

### Methods

The 3-parameter Weibull function was utilized to predict diameter distributions of the 4 conifer plantations based on the PSPs. This function and its CDF type are defined in equations (1) and (2) (Bailey and Dell 1973):

$$f(x) = \left(\frac{c}{b}\right) \left[\frac{(x-a)}{b}\right]^{c-1} \exp\left\{-\left[\frac{(x-a)}{b}\right]^c\right\} \text{ and (1)}$$

$$F(x) = 1 - \exp\left\{-\left[\frac{(x-a)}{b}\right]^c\right\}; \quad (2)$$

where  $x$  is the DBH; and  $a$ ,  $b$ , and  $c$  are location, scale, and shape parameters of the Weibull function.

We adopted the MLE and PE to estimate parameters of the Weibull function. In general, when using the MLE to estimate the Weibull function, one needs to determine the location parameter,  $a$ , at the beginning stage of the process (Bailey 1974). This parameter was estimated based on a method proposed by Zanakis (1979) and is defined as:

$$\hat{a} = \frac{x_1 x_n - x_2^2}{x_1 + x_n - 2x_2}; \quad (3)$$

where  $x_i$  is the  $i$ th ordered value in the plot, and  $n$  is the number of trees.

As the parameter  $a$  is determined, we should calculate each DBH -  $\hat{a}$  of trees for stands and let  $x'_i = DBH_i - \hat{a}$ . After this transformation, the Weibull function can be regarded as a 2-parameter Weibull function. The process of the MLE needs to define the maximum likelihood equations ( $L$ ) as equation (4), then take the logarithm of equation (4), and differentiate with respect to  $b$  and  $c$ , respectively, as equations (5) and (6). (Bailey and Dell 1973, Bailey 1974, Cao 2004, Lei 2008):

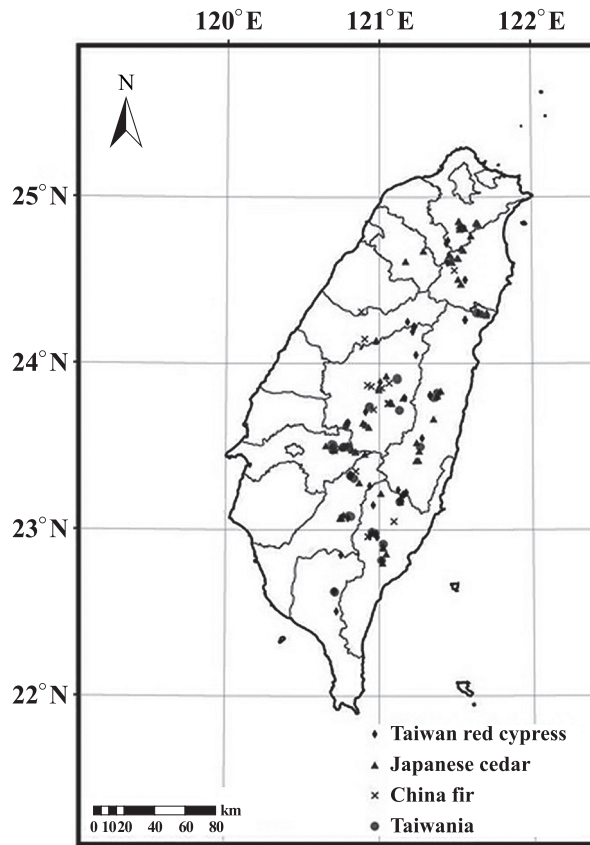


Fig. 1. Distribution of samples of 4 conifers examined in this study (Huang et al. 2011).

$$L(x'; b, c) = \prod_{i=1}^n f(x'_i; b, c) = \prod_{i=1}^n \frac{c}{b} \left(\frac{x'_i}{b}\right)^{c-1} \exp\left[-\left(\frac{x'_i}{b}\right)^c\right], \quad (4)$$

$$\frac{\partial \ln L(x'; b, c)}{\partial b} = 0, \text{ and} \quad (5)$$

$$\frac{\partial \ln L(x'; b, c)}{\partial c} = 0; \quad (6)$$

where  $x'_i$  is the  $i$ th ordered value of DBH-  $\hat{a}$ ,  $\hat{a}$  is defined as in equation (3), and  $b$  and  $c$  are parameters of the Weibull function.

Parameters  $b$  and  $c$  can be solved from equations (5) and (6) according to the Newton–Raphson method. For the detailed process of the MLE, please refer to Bailey (1974), Cao (2004), and Lei (2008).

On the other hand, the PE is based on special percentages of the CDF of the stand diameter distributions to predict the Weibull function, and 2 kinds of PE methods were used in this study, namely PE(I) and PE(II). The former adopts the 24<sup>th</sup>, 63<sup>rd</sup>, and 93<sup>rd</sup> percentiles of the stand diameter distributions ( $X_{.24}$ ,  $X_{.63}$ , and  $X_{.93}$ ) as a basis to recover parameters of the Weibull function, and this approach was proposed by Abernethy (1981). The latter only uses the 24<sup>th</sup> and 93<sup>rd</sup> percentiles subjected to the location parameter which is obtained, and these 2 percentages were proposed by Dubey (1967). PE(II) determines the location parameter the same as in equation (3). According to the CDF of the Weibull

function as shown in equation (2), relationships between the  $p$ -percentile value of the diameter distributions  $X_p$  and parameters of the Weibull function can be expressed as  $X_p = a + b \{-\ln(1-p)\}^{1/c}$ , where  $p$  represents the  $p$ -percentile of the diameter distributions, and  $a$ ,  $b$ , and  $c$  are parameters of the Weibull function (Clutter et al. 1983). Therefore, DBH at the 24<sup>th</sup>, 63<sup>rd</sup>, and 93<sup>rd</sup> percentiles of the stand diameter distributions can be expressed by the following equations (Clutter et al. 1983):

$$X_{24} = a + b \{-\ln(.76)\}^{1/c}, \quad (7)$$

$$X_{63} = a + b \{-\ln(.37)\}^{1/c} \approx a + b, \quad \text{and} \quad (8)$$

$$X_{93} = a + b \{-\ln(.07)\}^{1/c}; \quad (9)$$

where  $X_p$  is the DBH at the  $p$ -percentile of the diameter distributions, and  $a$ ,  $b$ , and  $c$  are parameters of the Weibull function.

Since  $\ln(.37)$  is approximately -1, equation (5) can be simplified to  $a + b$ . In other words, the  $b$  value is revealed to be approximately equal to the 63<sup>rd</sup> percentile of the stand diameter distributions when  $a = 0$  (Bailey and Dell 1973, Clutter et al. 1983). The PE(I) solution of parameters  $a$ ,  $b$ , and  $c$  is based on equations (7)~(9). Parameter  $c$  can be solved iteratively by the formula of  $(X_{24} - X_{63}) / (X_{93} - X_{63}) = [\text{equation (7) - equation (8)}] / [\text{equation (9) - equation (8)}]$ . Once the  $c$  value is determined, the  $b$  value can be solved by equations (7)~(9), and then the  $a$  value is calculated as  $a = X_{63} - b$  (Clutter et al. 1983). However, PE(II) only uses equations (7) and (9) to solve parameters  $b$  and  $c$  when parameter  $a$  is determined as in equations (3). Parameter  $c$  can be solved based on equations (7) and (9) when parameter  $a$  is known. Once parameters  $a$  and  $c$  are obtained, parameter  $b$  can be solved from equations (7) and (9). PE(I) differs from PE(II) in the order of solving parameters, with the former beginning with parameter  $c$ , followed by parameters  $b$  and  $a$ ; while the latter begins with parameter  $a$ , followed by parameters  $c$  and  $b$ . According to these rules,

parameters of the Weibull function were obtained for PE(I) and PE(II), and for the detailed processes, please refer to Dubey (1967), Abernethy (1981), and Clutter et al. (1983).

A goodness-of-fit test for the observed data and the Weibull distribution used the Kolmogorov-Smirnov (K-S) test to examine when the parameters were estimated from the data. The K-S statistic is based on the CDF of the data and the Weibull distribution and calculates its maximum differences as the criterion for examination (Sokal and Rohlf 1981). Because the Weibull distribution was predicted using 3 approaches (the MLE, PE(I), and PE(II)), each sample needs to compare the Weibull distributions with different approaches. We adopted  $\alpha = 0.05$  as a significance level for the K-S test for all samples.

## RESULTS AND DISCUSSION

### Stand characteristics of samples for the 4 conifers

We found higher values for Japanese cedar and lower values for Taiwan red cypress for most stand characteristics except for the tree number  $\text{ha}^{-1}$  compared to the other species (Table 1). As to the special percentiles of DBH, the order was  $X_{93} > X_{63} > X_{24}$  for all species. Because  $X_{24}$ ,  $X_{63}$ , and  $X_{93}$  were calculated from the DBH at the 24<sup>th</sup>, 63<sup>rd</sup>, and 93<sup>rd</sup> percentiles of the diameter distributions, it was expected that the larger percentiles possessed larger values of the DBH. Moreover, we also found that values of the mean DBH were lower than the 63<sup>rd</sup> diameter distribution ( $X_{63}$ ), regardless of species.

### Parameters estimated from the 3 approaches for the 4 species

We estimated the parameters of the Weibull function with different approaches and found that the parameters varied with



**Table 1. Stand characteristics of sample number, age, mean diameter at breast height (DBH), mean tree height, and the 24<sup>th</sup>, 63<sup>rd</sup>, and 93<sup>rd</sup> percentiles (X) of the diameter distributions for 4 conifer species**

Stand characteristics	Species			
	China fir	Japanese cedar	Taiwan red cypress	Taiwania
Sample number	18	88	55	36
Age (yr)	32.2 ± 10.9 <sup>1)</sup>	41.7 ± 13.3	23.3 ± 7.7	26.0 ± 8.0
Tree number (stems ha <sup>-1</sup> )	1345 ± 875	1146 ± 586	1122 ± 478	1268 ± 641
Mean DBH (cm)	21.1 ± 5.2	28.8 ± 7.8	19.2 ± 4.4	22.0 ± 5.5
Mean tree height (m)	11.4 ± 2.2	16.1 ± 3.3	9.7 ± 2.6	11.8 ± 3.6
Basal area (m <sup>2</sup> ha <sup>-1</sup> )	45.8 ± 27.1	73.3 ± 33.8	35.3 ± 18.9	54.9 ± 33.7
X.24	16.7 ± 4.1	22.8 ± 7.0	15.0 ± 3.9	17.2 ± 5.0
X.63	22.7 ± 5.7	31.2 ± 8.4	20.6 ± 4.8	24.0 ± 6.2
X.93	29.1 ± 7.8	40.5 ± 10.9	26.7 ± 5.8	31.4 ± 8.3

<sup>1)</sup> Mean ± standard deviation.

the approaches except for parameter *a* with the MLE and PE(II) approaches (Table 2). Because the same method [equation (3)] was used to estimate parameter *a*, the same results were obtained. The parameters imply a special meaning for the Weibull distribution where parameters *a*, *b*, and *c* indicate the initial location (the smallest tree), scale, and shape, respectively (Bailey and Dell 1973). Higher values of parameter *a* were found with PE(I) for China fir and Japanese cedar compared to the other approaches (Table 2). We found the smallest *b* values for Japanese cedar, Taiwan red cypress, and Taiwania with the MLE approach, however for China fir it was with the PE(I) approach. Bailey and Dell (1973) pointed out that parameter *c* determines the shape of the Weibull distribution, and the distribution types show positively skewed distributions when  $1 < c < 3.6$ . We found smaller *c* values with the MLE approach for all species. The range of *c* values was  $1 < c < 3.6$  for all species regardless of the approach, implying that most of the stand diameter distributions were positively skewed. Moreover, simulated samples were not equal among approaches for each species in Table 2, because all samples

could be simulated by the MLE and PE(II) approaches, but some samples could not be estimated by the PE(I) approach. Two main types were found in these failed samples for which parameter *c* of the samples could not successfully be solved by equations (7)~(9), and a negative value was estimated for parameter *a* of those samples. The process of solving parameters begins with parameter *c*, followed by parameter *b*, and then parameter *a* for PE(I). Because parameter *a* is influenced by the other parameters, this is probably why a negative value was obtained. However, parameter *a* indicates the smallest DBH of the stands (Bailey and Dell 1973). Therefore this parameter should be a positive value. If a negative value was estimated for certain samples, then those samples were eliminated in this step. The pass ratios of PE(I) varied with species; for instance, the respective pass ratios were 44.4, 60.0, 63.6, and 72.2% for China fir, Japanese cedar, Taiwan red cypress and Taiwania (Table 2).

### The K-S test for examining the 3 approaches for 4 species

We used the K-S test to examine diameter

**Table 2. Estimated parameters of the Weibull function in each permanent sample plot (PSP) from 3 approaches for 4 species**

Species	Approaches <sup>1)</sup>	Simulated samples <sup>2)</sup>	Parameters		
			a	b	c
China fir	MLE	18	9.39 ± 3.61 <sup>3)</sup>	10.33 ± 4.43	1.54 ± 0.70
	PE(I)	8	12.92 ± 3.99	8.86 ± 1.92	1.55 ± 0.58
	PE(II)	18	9.39 ± 3.61	12.72 ± 4.55	2.33 ± 0.70
Japanese cedar	MLE	88	12.26 ± 5.25	14.63 ± 8.13	1.88 ± 0.92
	PE(I)	51	13.54 ± 10.60	16.53 ± 8.79	2.17 ± 1.24
	PE(II)	88	12.26 ± 5.25	18.15 ± 8.39	2.35 ± 0.86
Taiwan red cypress	MLE	55	9.56 ± 4.49	8.25 ± 3.77	1.66 ± 0.72
	PE(I)	35	9.06 ± 5.64	10.62 ± 6.26	1.93 ± 0.87
	PE(II)	55	9.56 ± 4.49	10.28 ± 3.83	2.04 ± 0.72
Taiwania	MLE	36	9.16 ± 4.21	11.21 ± 5.37	1.68 ± 0.86
	PE(I)	26	8.47 ± 6.98	14.34 ± 7.28	2.72 ± 1.83
	PE(II)	36	9.16 ± 4.21	14.07 ± 5.94	2.37 ± 0.90

<sup>1)</sup> MLE, maximum likelihood estimator; PE(I), percentile estimator (I); and PE(II), percentile estimator (II).

<sup>2)</sup> Simulated samples indicate that the data were successfully predicted by the method. Because some samples failed to be predicted by the PE(I) approach, numbers of simulated samples are not equal for the same species.

<sup>3)</sup> Mean ± standard deviation.

distributions of stands with the 3 approaches (Table 3) and calculated the ratios of samples that passed to total samples as an indicator ( $R_{ps/ts}$ ) (Fig. 2). In general, the PE(II) approach was superior to the other approaches, and this approach had higher  $R_{ps/ts}$  values for all species; for instance, respective  $R_{ps/ts}$  values for China fir, Japanese cedar, Taiwan red cypress, and Taiwania were 100, 96.6, 85.5, and 97.2%. Ranges of  $R_{ps/ts}$  values were 33.3~44.3 and 44.4~63.9% for the MLE and PE(I) approaches for all species. We also found the same trend for each species, that is, the  $R_{ps/ts}$  of PE(II) > PE(I) > MLE.

In a previous study, site index curves were established for these 4 conifers based on the same database (Huang et al. 2011). The potential productivity of plantations can be assessed following this site index system, and in this study, we fit diameter distributions for these 4 conifers. Quantifying diameter

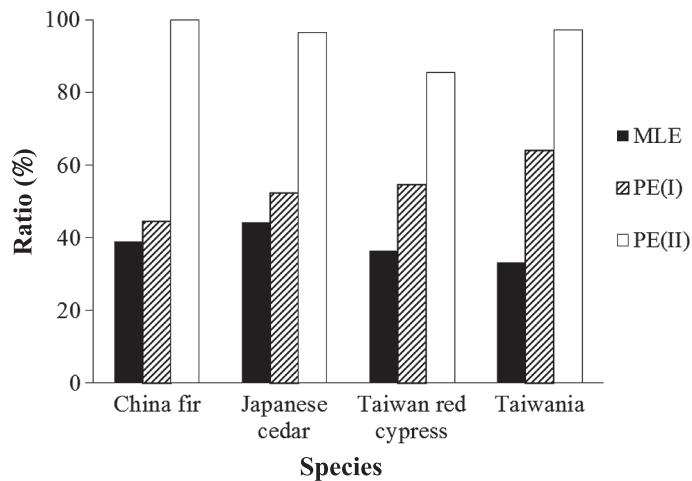
distributions of plantation forests can help in understanding the stand structure and managing forests. We also found that various statistical methods were adapted to examine the observed data and the Weibull distribution for forests, such as the K-S test, the Anderson-Darling statistic, and the error index (Cao 2004). Among those methods, the K-S test is widely utilized to test the Weibull distribution in forestry (e.g., Nanang 1998, Cao 2004, Li and Yen 2010), because the K-S test possesses many advantages, such as its simple performance and sensitivity when comparing differences in skewness and dispersion (Sokal and Rohlf 1981). Therefore, we chose the K-S test as a method to examine the observed data and the Weibull distribution estimated by the 3 approaches. From the results of this test, we found that 3 approaches can be obviously distinguished by the K-S test, and the PE(II) approach was suitable for the 4 conifer planta-



**Table 3. The Kolmogorov-Smirnov test for examining 3 approaches for 4 species**

Species	Method <sup>1)</sup>	Total samples	Examined samples	Dn		No. of samples that passed
				Mean	Range	
China fir	MLE	18	18	0.279	0.188~0.396	7
	PE(I)	18	8	0.126	0.085~0.201	8
	PE(II)	18	18	0.149	0.076~0.231	18
Japanese cedar	MLE	88	88	0.284	0.125~0.605	39
	PE(I)	88	51	0.114	0.047~0.194	46
	PE(II)	88	88	0.149	0.054~0.364	85
Taiwan red cypress	MLE	55	55	0.301	0.111~0.667	20
	PE(I)	55	35	0.138	0.064~0.347	30
	PE(II)	55	55	0.180	0.063~0.388	47
Taiwania	MLE	36	36	0.291	0.165~0.530	12
	PE(I)	36	26	0.131	0.092~0.170	23
	PE(II)	36	36	0.153	0.091~0.355	35

<sup>1)</sup>MLE, maximum likelihood estimator; PE(I), percentile estimator (I); and PE(II), percentile estimators (II).



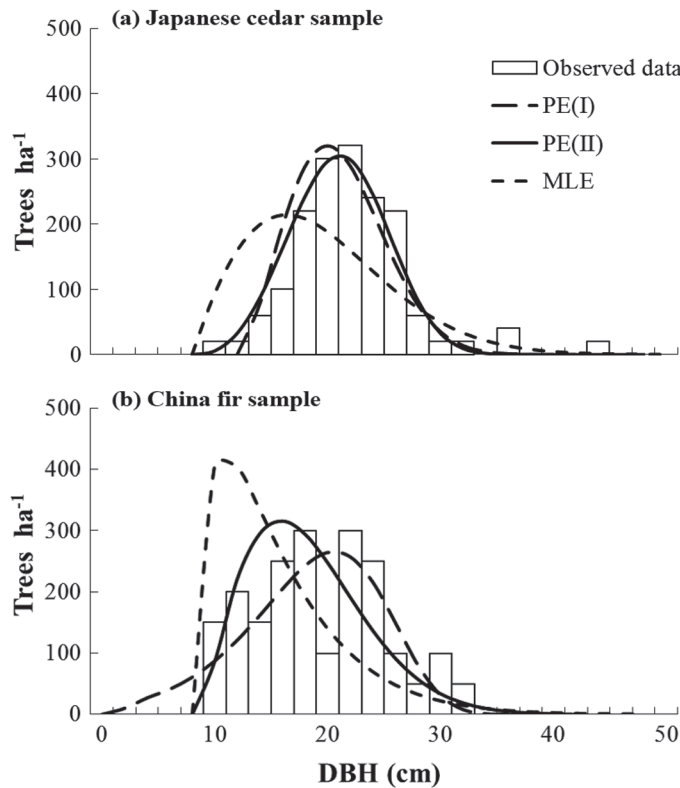
**Fig. 2. Ratios of samples that passed to total samples among the 3 approaches for each species. MLE, maximum likelihood estimator; PE, percentile estimator.**

tions compared to the other approaches. This approach will be taken into consideration when developing whole-stand models based on stand diameter distributions.

**Examples of the simulation effects for different approaches**

In order to show relationships between the observed data and the Weibull distribu-

tion with different approaches, 2 samples of Japanese cedar and China fir were used for illustration (Fig. 3). The first sample (Japanese cedar) showed that diameter distributions passed the K-S examination with the PE(I) and PE(II) approaches but failed to pass for the MLE approach. Parameters *a*, *b*, and *c* were respectively predicted to be 8.20, 12.59, and 1.83 for the MLE; 12.34, 9.79, and 2.29



**Fig. 3.** Two samples of Japanese cedar and China fir which illustrate the simulation effects between observed data and the Weibull distribution with different approaches, (a) diameter distributions of the Japanese cedar samples that passed the Kolmogorov-Smirnov (K-S) examination for the percentile estimator (PE(I) and PE(II) approaches); (b) among diameter distributions of China fir samples, for which only the PE(II) approach passed the K-S examination. DBH, diameter at breast height.

for the PE(I); and 8.20, 14.29, and 3.34 for the PE(II) approach (the significance level  $D_n = 0.1491$  for the K-S test). The second sample (China fir) showed that diameter distributions only passed the K-S examination with the PE(II) approach, while the other approaches failed. Parameters  $a$ ,  $b$ , and  $c$  were respectively predicted to be 9.18, 7.10, and 1.20 for the MLE; -44.74, 65.80, and 11.85 for the PE(I); and 9.18, 10.29, and 1.83 for the PE(II) approach (the significance level  $D_n = 0.2147$  for the K-S test). We also found that parameter  $a$  had a negative value when the PE(I) approach was used. This is unreasonable, because this

parameter indicates the DBH of the smallest tree. Therefore, we do not suggest the PE(I) approach when parameter  $a$  is a negative value.

We also found that many research studies focused on estimating the Weibull function with different approaches for diameters of stands worldwide. For instance, Zarnoch and Dell (1985) evaluated the MLE and PE as estimators to estimate the Weibull function for diameters of stands and found that the former had superior smaller biases but larger variances than the latter. They also pointed out that both of these approaches can adequately

estimate diameter distributions of pine plantations (Zarnoch and Dell 1985). Nanang (1998) assessed the MLE, PE, and moment estimators (MEs) to estimate the Weibull distribution for neem (*Azadirachia indica*) plantations and found that the MLE and ME approaches were superior to the PE approach after examination by the K-S test. Lei (2008) compared the MLE, ME, and least-squares regression method (LSM) to estimate the Weibull distribution for China pine (*Pinus tabulaeformis*) and used the mean square error and sample size as criteria to compare these methods. The results showed that the ME approach was superior to the others for estimating the Weibull distribution. However, our study revealed that the PE(II) approach was most suitable for the 4 conifer plantations. Overall, an interesting result was the variation in the suitability of approaches with different tree species.

## CONCLUSIONS

The importance of plantations arises from current forest management practices because many environmental, ecological, and economic targets are widely programmed in plantation forests. A large number of man-made forests of approximately 420,000 ha are distributed around Taiwan. Among these man-made forests, China fir, Japanese cedar, Taiwan red cypress, and Taiwania are 4 important conifers and are widely planted in mountain areas with good economic and ecological values. We also found that numerous research studies focused on growth and yield estimates of these 4 conifers, especially on a regional scale, whereas, few studies addressed larger scales. Quantifying diameter distributions of stands is a valid expression to show distributions of tree sizes and can provide fundamental information for different aspects of forest management. The Weibull function is

widely utilized to fit diameter distributions of stands, and their simulation effects may vary with different approaches. This study used the Weibull function to estimate diameter distributions for the 4 conifers based on PSP data, and these PSPs were distributed throughout Taiwan. Three approaches were chosen to estimate the parameters of the Weibull function for these 4 conifer plantations, and we found that the PE(II) approach was superior to the other approaches after examining the goodness-of-fit using the K-S test. Respective pass rates of samples were 100, 96.6, 85.5, and 97.2% for China fir, Japanese cedar, Taiwan red cypress, and Taiwania. This indicates that the PE(II) approach is the best estimator, regardless of species, and this approach will be taken into consideration when developing whole-stand models based on stand diameter distributions in the future.

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